The Linear Optical Unambiguous Discrimination of Hyperentangled Bell States Assisted by Time Bin

Cheng-Yan Gao, Bao-Cang Ren, Yuan-Xia Zhang, Qing Ai, and Fu-Guo Deng*

A linear optical unambiguous discrimination of hyperentangled Bell states is proposed for two-photon systems entangled in both the polarization and momentum degrees of freedom (DOFs) assisted by time bin. This unambiguous discrimination scheme can completely identify 16 orthogonal hyperentangled Bell states using only linear optical elements, where the function of the auxiliary entangled Bell state is replaced by time bin. Moreover, the possibility of extending this scheme for distinguishing hyperentangled Bell states in n DOFs is discussed, and it shows that 2^{n+k+1} hyperentangled Bell states in n ($n \geq 2$) DOFs can be distinguished with k ($k < n$) auxiliary entangled states of additional DOFs by introducing a time delay, which decreases the auxiliary entanglement resource required for unambiguous discrimination of hyperentangled Bell state. Therefore, this scheme provides a new way for distinguishing hyperentangled states with current technology, which will extend the application of discrimination of hyperentangled states via linear optics to other quantum information protocols besides hyperdense coding schemes in the future.

1. Introduction

Photon system is one of the most interesting candidates for quantum communication with the character of manipulability, high-speed transmission, and high capacity. The entangled photon system is a crucial quantum resource in quantum communication, which has many important applications in quantum communication protocols, such as quantum key distribution,[1–4] quantum secret sharing,[5] quantum dense coding,[6,7] quantum teleportation,[8] quantum secure direct communication,[9–12] and so on. In these schemes, the discrimination of quantum states is a fundamental task in quantum communication.[13–17] Especially, the deterministic distinction of photonic entangled states is necessary and indispensable to read out the quantum information. With linear optical elements, it is difficult to unambiguously identify the orthogonal entangled states of photon system. For example, using linear optical elements, the probabilistic Bell-state analysis (BSA) with 50% efficiency can be attained both in theory[18–20] and in experiment.[21–23]

In fact, the photonic qubits can be encoded in several degrees of freedom (DOFs) of photon systems, such as the polarization, momentum, temporal, frequency, and so on. The entanglement in more than one DOF, called hyperentanglement, has been demonstrated in experiment for different DOFs of photon systems, such as the photon systems simultaneously entangled in polarization–frequency DOFs,[24] polarization–temporal DOFs,[25] polarization–momentum DOFs,[26–28] polarization–momentum–temporal DOFs,[29] and polarization-orbital-angular-momentum DOFs.[30] With the help of further DOFs, the deterministic complete two-photon BSA can be implemented by using linear optical elements. In 1998, Kwiat and Weinfurter[31] first showed that all of the four two-photon polarization Bell states could be unambiguously identified with linear optical elements, assisted by the time–energy correlation of photon pair occurring in the spontaneous parametric down-conversion (SPDC) process or the entanglement of momentum DOF. In 2003, Walborn et al.[32] proposed a simple scheme for complete Bell-state measurement of photon system with polarization-momentum hyperentangled state, where the four polarization (momentum) Bell states can be distinguished completely by using momentum (polarization) entanglement as an auxiliary. In 2006, Schuck et al.[25] gave the complete analysis of the four polarization Bell states in experiment with the help of the intrinsic time–energy correlation of photon pair. In 2007, a deterministic Bell-state measurement was experimentally demonstrated by Barbieri et al.[26] with two-photon polarization-momentum hyperentanglement via linear optics. Recently, Williams et al.[33] demonstrated an attractive protocol in experiment to completely differentiate the four polarization Bell states assisted by two time delays, where only

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linear optical elements and common single-photon detectors are required.

Besides the applications in conventional BSA, hyperentanglement can offer significant advantages in other quantum communication protocols.[34] Hyperentanglement has important applications in quantum error-correcting,[35,36] quantum cryptography,[37,38] quantum repeater,[39] and entanglement purification.[40–43] Moreover, hyperentanglement can be used to enhance the channel capacity of quantum communication, such as in hyperdense coding,[30] quantum hyperteleportation,[44,45] hyperentanglement swapping,[46] hyperentanglement purification,[47–49] and hyperentanglement concentration,[50,51] and it can also be used to reduce the operation time and the resource consumption by constructing hyperparallel photonic quantum computation.[52–55] In high-capacity quantum communication, hyperentangled Bell-state analysis (HBSA) is one of the most popular components to read out the quantum information. It has been shown that, with linear optics, 7 groups can be formed for the 16 orthogonal hyperentangled Bell states, and the upper bound of the distinguishable groups for a system entangled in n DOFs is $(2^{n+1}-1)$.[56,57] With an auxiliary entangled state of another momentum DOF, the 16 hyperentangled Bell states in the polarization and momentum DOFs can be classified into 12 groups via linear optics, and 4^n hyperentangled Bell states for a system entangled in n DOFs can be separated into $(2^{n+k+1}-2^{2k})$ groups with the help of k ($k \leq n$) ancillary entangled states of other DOFs.[58] With nonlinear optical elements, the complete HBSA can be constructed in theory.[44,46,59–63] In 2010, Sheng et al.[44] presented the first complete HBSA with cross-Kerr nonlinearity, in which the entanglement in different DOFs is manipulated independently. In 2012, Ren et al.[46] and Wang et al.[59] proposed the schemes for the complete differentiation of 16 hyperentangled Bell states in both polarization and momentum DOFs by using one-sided quantum-dot cavity (QD-cavity) system and double-sided QD-cavity system, respectively. Later, some interesting complete HBSA protocols via nonlinear optics were proposed.[60–63] The HBSA scheme with auxiliary entangled states will limit its application to the entangled photon system (such as in hyperdense coding), which is not suitable for hyperteleportation and hyperentanglement swapping, and the realization of high-fidelity and high-efficiency photon interaction with nonlinear optical elements remains a hard problem to be overcome with the current technology.

In this article, we unambiguously distinguish the 16 orthogonal hyperentangled Bell states in both the polarization and momentum DOFs via linear optics assisted by time bin and an auxiliary fixed Bell state of another momentum DOF, where the function of the other auxiliary entangled Bell state used in previous works is replaced by time bin. In this scheme, the parity information of the momentum Bell state can be distinguished by introducing a time delay, and the parity information of the polarization Bell state can be distinguished assisted by an auxiliary fixed Bell state of another momentum DOF. The phase information of the polarization Bell state and momentum Bell state can be distinguished using the product measurement. Moreover, we discuss the possibility of extending this scheme for distinguishing hyperentangled Bell states in n ($n \geq 2$) DOFs, and we have come to the conclusion that 2^{n+k+1} hyperentangled Bell states in n ($n \geq 2$) DOFs can be distinguished with k ($k < n$) auxiliary entangled states of additional DOFs by introducing a time delay, which means more hyperentangled Bell states can be identified exactly compared with the previous works. As the time bin is introduced in the unambiguous discrimination scheme instead of the entanglement initially prepared in photon system, it provides a new way for distinguishing hyperentangled state with current technology, and its application will be extended to other quantum information protocols (e.g. hyperteleportation and hyperentanglement swapping) besides hyperdense coding scheme in the future.

2. The Linear Optical Unambiguous Discrimination of Hyperentangled Bell State Assisted by Time Bin

Here we consider a two-photon system simultaneously entangled in both the polarization DOF and momentum DOF named as $\vec{k}$,[27] which can be directly prepared using a single suitably chosen down-conversion crystal and linear optical elements.[28] The hyperentangled two-photon state could be one of 16 hyperentangled Bell states, which results from the combination of the four polarization Bell states and the four momentum Bell states ($|\varphi_P^{\pm}\rangle \otimes |\varphi_S^{\pm}\rangle$). Here, the four polarization Bell states are

$$|\phi_P^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle)_{AB}$$
$$|\psi_P^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|HV\rangle \pm |VH\rangle)_{AB} \tag{1}$$

And the four momentum Bell states are

$$|\phi_S^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|II\rangle \pm |EE\rangle)_{AB}$$
$$|\psi_S^{\pm}\rangle_{AB} = \frac{1}{\sqrt{2}}(|IE\rangle \pm |EI\rangle)_{AB} \tag{2}$$

The subscripts A and B represent the two photons in the hyperentangled system. The subscripts P and S denote the polarization DOF and momentum DOF, respectively. $|H\rangle$ and $|V\rangle$ are the horizontal and vertical polarization states, respectively. $|I\rangle$ and $|E\rangle$ belong to the basis of the momentum DOF $\vec{k}$, and they denote the internal state ($|I\rangle$) and the external state ($|E\rangle$), respectively. ϕ and ψ ("+" and "−") represent the parity mode (phase mode) in the polarization and momentum DOFs of two-photon system.

In order to distinguish the 16 hyperentangled Bell states, a time delay and an additional entangled state of another momentum DOF for two-photon system AB labeled as $\vec{c}$[27] are utilized. The time delay is introduced during the analysis process of hyperentangled Bell state, and the auxiliary entangled state is introduced in the preparation process of hyperentangled Bell state, which can be expressed as

$$|\chi_S^{+}\rangle_{AB} = \frac{1}{\sqrt{2}}(|lr\rangle + |rl\rangle)_{AB} \tag{3}$$

Here $|l\rangle$ and $|r\rangle$ are regarded as the left and right states of the momentum DOF $\vec{c}$, respectively. In experiment, the hyperentangled Bell state in the polarization and two momentum DOFs

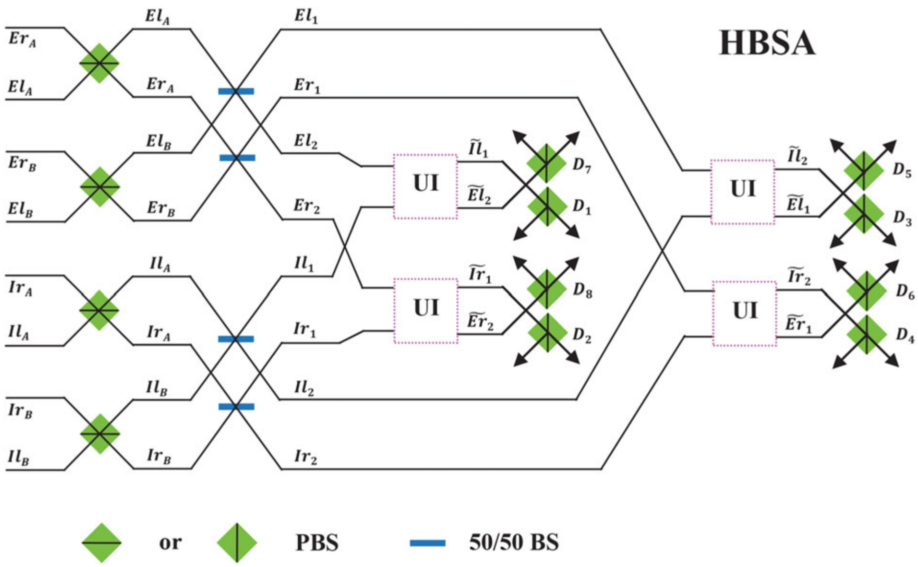

Figure 1. Schematic diagram for the linear optical complete HBSA assisted by time bin. PBS represents a polarizing beam splitter, which transmits the horizontally polarized component $|H\rangle$ and reflects the vertically polarized component $|V\rangle$ of photons. BS denotes the 50:50 beam splitter. UI denotes the unbalanced interferometer in Figure 2. The output mode is detected on the polarization basis $\{|H\rangle, |V\rangle\}$ with PBS.

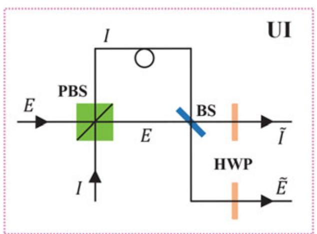

Figure 2. Schematic diagram of the unbalanced interferometer (UI). HWP represents a half-wave plate which can perform the Hadamard operation on the polarization DOF. The optical loop on the path I denotes an optical time delay t.

$(|\varphi_P^{\pm}\rangle \otimes |\varphi_S^{\pm}\rangle \otimes |\chi_S^{+}\rangle)$ has been generated by using a two-crystal system constituted of two type I β barium borate BBO crystal slabs which are aligned one behind the other.[27]

The setup of the complete HBSA assisted by time bin and momentum DOF $\vec{c}$ is shown in **Figure 1**, where UI is a very important element for introducing time bin (shown in **Figure 2**). The UI is composed of a polarizing beam splitter (PBS), a 50:50 beam splitter (BS), and two half-wave plates (HWPs). The PBS is used to transmit the horizontal polarization state $|H\rangle$ and reflect the vertical polarization state $|V\rangle$. After passing through PBS, the wave packets of photons in the long arm I of UI will get the optical time delay t. BS is used to perform unitary transformation on the momentum DOF. HWP represents a half-wave plate with the angle 22.5° to the horizontal direction, which can perform Hadamard operation on the polarization DOF.

In the following, we take the analysis of hyperentangled Bell state $|\psi_P^+\psi_S^-\chi_S^+\rangle_{AB}$ as an example to explain the principle of complete HBSA in detail. The hyperentangled Bell state $|\psi_P^+\psi_S^-\chi_S^+\rangle_{AB}$ for two-photon system AB can be expressed as

$$|\Theta_1^0\rangle_{AB} = \frac{1}{2\sqrt{2}}(|HV\rangle + |VH\rangle)_{AB} \otimes (|IE\rangle - |EI\rangle)_{AB} \otimes(|lr\rangle + |rl\rangle)_{AB} \tag{4}$$

First, the two photons A and B are sent into PBSs as shown in Figure 1. After passing through PBSs, the hyperentangled Bell state of two-photon system AB becomes

$$|\Theta_1^1\rangle_{AB} = \frac{1}{2\sqrt{2}}(|HV\rangle + |VH\rangle) \otimes [|Il_A El_B\rangle + |Ir_A Er_B\rangle - |El_A Il_B\rangle - |Er_A Ir_B\rangle] \tag{5}$$

Subsequently, the two photons A and B are sent into BSs shown in Figure 1, where the unitary transformations on the momentum DOF are performed in the following forms[64,65]

$$\begin{aligned} X_{AH}^{\dagger} &\rightarrow \frac{1}{\sqrt{2}}(X_{1H}^{\dagger} + X_{2H}^{\dagger}), \quad X_{AV}^{\dagger} \rightarrow \frac{1}{\sqrt{2}}(X_{1V}^{\dagger} + X_{2V}^{\dagger}) \\ X_{BH}^{\dagger} &\rightarrow \frac{1}{\sqrt{2}}(X_{1H}^{\dagger} - X_{2H}^{\dagger}), \quad X_{BV}^{\dagger} \rightarrow \frac{1}{\sqrt{2}}(X_{1V}^{\dagger} - X_{2V}^{\dagger}) \end{aligned} \tag{6}$$

Here X denotes one of the symbols Il, Ir, El, and Er. Correspondingly, $X_{AH}^{\dagger}$, $X_{BH}^{\dagger}$, $X_{AV}^{\dagger}$, and $X_{BV}^{\dagger}$ are the creation operators of the input modes of the beam splitter, and $X_{1H}^{\dagger}$, $X_{2H}^{\dagger}$, $X_{1V}^{\dagger}$, and $X_{2V}^{\dagger}$ are the creation operators of the output modes of the beam splitter. After the two photons pass through BSs, the hyperentangled state $|\Theta_1^1\rangle_{AB}$ evolves to

$$\begin{aligned} |\Theta_1^2\rangle_{AB} = \frac{1}{2\sqrt{2}}(&-Il_{1H}^{\dagger}El_{2V}^{\dagger} + Il_{2H}^{\dagger}El_{1V}^{\dagger} - Ir_{1H}^{\dagger}Er_{2V}^{\dagger} \\ &+Ir_{2H}^{\dagger}Er_{1V}^{\dagger} + El_{1H}^{\dagger}Il_{2V}^{\dagger} - El_{2H}^{\dagger}Il_{1V}^{\dagger} \\ &+Er_{1H}^{\dagger}Ir_{2V}^{\dagger} - Er_{2H}^{\dagger}Ir_{1V}^{\dagger})|0\rangle \end{aligned} \tag{7}$$

The above process can only readout the parity information of polarization DOF, and eight hyperentangled Bell states can be unambiguously distinguished after two-photon product measurement.[58] To get the parity information of momentum DOF $\vec{k}$ and enable the complete HBSA, we introduce the time delay in UI (the schematic diagram is shown in Figure 2) before the two-photon product measurement. That is, the wave packets in the momentum modes $Il_1^\dagger|0\rangle$, $Il_2^\dagger|0\rangle$, $Ir_1^\dagger|0\rangle$, and $Ir_2^\dagger|0\rangle$ are sent into UI from the port I of PBS, which are either transmitted to experience a time delay t when the polarization mode is $|H\rangle$ or reflected without time delay when the polarization mode is $|V\rangle$. The wave packets in the momentum modes $El_1^\dagger|0\rangle$, $El_2^\dagger|0\rangle$, $Er_1^\dagger|0\rangle$, and $Er_2^\dagger|0\rangle$ are sent into UI from the port E of PBS, which are either reflected to experience a time delay t when the polarization mode is $|V\rangle$ or transmitted without time delay when the polarization mode is $|H\rangle$. After the wave packets pass through PBSs and experience the time delay t, the hyperentangled state $|\Theta_1^2\rangle_{AB}$ evolves to

$$\begin{aligned}|\Theta_1^3\rangle_{AB} = \frac{1}{2\sqrt{2}}(&-Il_{1H'}^\dagger Il_{1V'}^\dagger + Il_{2H'}^\dagger Il_{2V'}^\dagger - Ir_{1H'}^\dagger Ir_{1V'}^\dagger\\ &+Ir_{2H'}^\dagger Ir_{2V'}^\dagger + El_{1H}^\dagger El_{1V}^\dagger - El_{2H}^\dagger El_{2V}^\dagger\\ &+Er_{1H}^\dagger Er_{1V}^\dagger - Er_{2H}^\dagger Er_{2V}^\dagger)|0\rangle\end{aligned} \tag{8}$$

Here H' (V') represents the component that experiences a time delay t. Then the two photons are sent into BSs shown in Figure 2. The creation operators of the input modes and the output modes of BSs transform as follows:

$$\begin{aligned}I_{1H}^\dagger &\rightarrow \frac{1}{\sqrt{2}}(\widetilde{I}_{1H}^\dagger + \widetilde{E}_{2H}^\dagger), & I_{2H}^\dagger &\rightarrow \frac{1}{\sqrt{2}}(\widetilde{I}_{2H}^\dagger + \widetilde{E}_{1H}^\dagger)\\ I_{1V}^\dagger &\rightarrow \frac{1}{\sqrt{2}}(\widetilde{I}_{1V}^\dagger + \widetilde{E}_{2V}^\dagger), & I_{2V}^\dagger &\rightarrow \frac{1}{\sqrt{2}}(\widetilde{I}_{2V}^\dagger + \widetilde{E}_{1V}^\dagger)\\ E_{1H}^\dagger &\rightarrow \frac{1}{\sqrt{2}}(\widetilde{I}_{2H}^\dagger - \widetilde{E}_{1H}^\dagger), & E_{2H}^\dagger &\rightarrow \frac{1}{\sqrt{2}}(\widetilde{I}_{1H}^\dagger - \widetilde{E}_{2H}^\dagger)\\ E_{1V}^\dagger &\rightarrow \frac{1}{\sqrt{2}}(\widetilde{I}_{2V}^\dagger - \widetilde{E}_{1V}^\dagger), & E_{2V}^\dagger &\rightarrow \frac{1}{\sqrt{2}}(\widetilde{I}_{1V}^\dagger - \widetilde{E}_{2V}^\dagger)\end{aligned} \tag{9}$$

Here the auxiliary momentum modes ($|l\rangle$ and $|r\rangle$) are omitted for the sake of simplicity. With the effect of BSs, the hyperentangled state $|\Theta_1^3\rangle_{AB}$ evolves to

$$\begin{aligned}|\Theta_1^4\rangle_{AB} = \frac{1}{4\sqrt{2}}[&-(\widetilde{Il}_{1H'}^\dagger + \widetilde{El}_{2H'}^\dagger)(\widetilde{Il}_{1V'}^\dagger + \widetilde{El}_{2V'}^\dagger)\\ &+(\widetilde{Il}_{2H'}^\dagger + \widetilde{El}_{1H'}^\dagger)(\widetilde{Il}_{2V'}^\dagger + \widetilde{El}_{1V'}^\dagger)\\ &-(\widetilde{Ir}_{1H'}^\dagger + \widetilde{Er}_{2H'}^\dagger)(\widetilde{Ir}_{1V'}^\dagger + \widetilde{Er}_{2V'}^\dagger)\\ &+(\widetilde{Ir}_{2H'}^\dagger + \widetilde{Er}_{1H'}^\dagger)(\widetilde{Ir}_{2V'}^\dagger + \widetilde{Er}_{1V'}^\dagger)\\ &+(\widetilde{Il}_{2H}^\dagger - \widetilde{El}_{1H}^\dagger)(\widetilde{Il}_{2V}^\dagger - \widetilde{El}_{1V}^\dagger)\\ &-(\widetilde{Il}_{1H}^\dagger - \widetilde{El}_{2H}^\dagger)(\widetilde{Il}_{1V}^\dagger - \widetilde{El}_{2V}^\dagger)\\ &+(\widetilde{Ir}_{2H}^\dagger - \widetilde{Er}_{1H}^\dagger)(\widetilde{Ir}_{2V}^\dagger - \widetilde{Er}_{1V}^\dagger)\\ &-(\widetilde{Ir}_{1H}^\dagger - \widetilde{Er}_{2H}^\dagger)(\widetilde{Ir}_{1V}^\dagger - \widetilde{Er}_{2V}^\dagger)]|0\rangle\end{aligned} \tag{10}$$

Here the wave packets with identical momentum and polarization modes but different time delays (e.g., the components $-\widetilde{Il}_{1H'}^\dagger\widetilde{El}_{2V'}^\dagger|0\rangle$ and $\widetilde{Il}_{1H}^\dagger\widetilde{El}_{2V}^\dagger|0\rangle$, or the components $\widetilde{Il}_{1H'}^\dagger\widetilde{Il}_{1V'}^\dagger|0\rangle$ and $\widetilde{Il}_{1H}^\dagger\widetilde{Il}_{1V}^\dagger|0\rangle$) can interfere destructively or constructively with each other ($e.g.$, $-\widetilde{Il}_{1H'}^\dagger\widetilde{El}_{2V'}^\dagger|0\rangle + \widetilde{Il}_{1H}^\dagger\widetilde{El}_{2V}^\dagger|0\rangle \rightarrow 0$, $\widetilde{Il}_{1H'}^\dagger\widetilde{Il}_{1V'}^\dagger|0\rangle + \widetilde{Il}_{1H}^\dagger\widetilde{Il}_{1V}^\dagger|0\rangle \rightarrow 2\widetilde{Il}_{1H}^\dagger\widetilde{Il}_{1V}^\dagger|0\rangle$). After the interference, the hyperentangled state $|\Theta_1^4\rangle_{AB}$ becomes

$$\begin{aligned}|\Theta_1^5\rangle_{AB} = \frac{1}{2\sqrt{2}}(&-Il_{1H}^\dagger Il_{1V}^\dagger - El_{2H}^\dagger El_{2V}^\dagger + Il_{2H}^\dagger Il_{2V}^\dagger\\ &+El_{1H}^\dagger El_{1V}^\dagger - Ir_{1H}^\dagger Ir_{1V}^\dagger - Er_{2H}^\dagger Er_{2V}^\dagger\\ &+Ir_{2H}^\dagger Ir_{2V}^\dagger + Er_{1H}^\dagger Er_{1V}^\dagger)|0\rangle\end{aligned} \tag{11}$$

In order to ensure the interference on BS occurs, the time delay t must satisfy the relation of constructive interference $\omega t = 2m\pi$ (m is an integer and ω is the frequency of input photons).[33]

Ultimately, HWPs perform the Hadamard operations on the polarization modes of the two photons. That is, $|H\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|V\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. After the two photons pass through HWPs, the hyperentangled state $|\Theta_1^5\rangle_{AB}$ is transformed to

$$\begin{aligned}|\Theta_1^6\rangle_{AB} = \frac{1}{4}(&-\widetilde{Il}_{1H}^\dagger\widetilde{Il}_{1H}^\dagger + \widetilde{Il}_{1V}^\dagger\widetilde{Il}_{1V}^\dagger - \widetilde{El}_{2H}^\dagger\widetilde{El}_{2H}^\dagger\\ &+\widetilde{El}_{2V}^\dagger\widetilde{El}_{2V}^\dagger + \widetilde{Il}_{2H}^\dagger\widetilde{Il}_{2H}^\dagger - \widetilde{Il}_{2V}^\dagger\widetilde{Il}_{2V}^\dagger\\ &+\widetilde{El}_{1H}^\dagger\widetilde{El}_{1H}^\dagger - \widetilde{El}_{1V}^\dagger\widetilde{El}_{1V}^\dagger - \widetilde{Ir}_{1H}^\dagger\widetilde{Ir}_{1H}^\dagger\\ &+\widetilde{Ir}_{1V}^\dagger\widetilde{Ir}_{1V}^\dagger - \widetilde{Er}_{2H}^\dagger\widetilde{Er}_{2H}^\dagger + \widetilde{Er}_{2V}^\dagger\widetilde{Er}_{2V}^\dagger\\ &+\widetilde{Ir}_{2H}^\dagger\widetilde{Ir}_{2H}^\dagger - \widetilde{Ir}_{2V}^\dagger\widetilde{Ir}_{2V}^\dagger + \widetilde{Er}_{1H}^\dagger\widetilde{Er}_{1H}^\dagger\\ &-\widetilde{Er}_{1V}^\dagger\widetilde{Er}_{1V}^\dagger)|0\rangle\end{aligned} \tag{12}$$

The hyperentangled state $|\Theta_1^6\rangle_{AB}$ can also be described in the following form

$$\begin{aligned}|\Theta_1^6\rangle_{AB} = \frac{1}{4}(&-|HH\rangle|\widetilde{Il}_1\widetilde{Il}_1\rangle + |VV\rangle|\widetilde{Il}_1\widetilde{Il}_1\rangle\\ &-|HH\rangle|\widetilde{El}_2\widetilde{El}_2\rangle + |VV\rangle|\widetilde{El}_2\widetilde{El}_2\rangle\\ &+|HH\rangle|\widetilde{Il}_2\widetilde{Il}_2\rangle - |VV\rangle|\widetilde{Il}_2\widetilde{Il}_2\rangle\\ &+|HH\rangle|\widetilde{El}_1\widetilde{El}_1\rangle - |VV\rangle|\widetilde{El}_1\widetilde{El}_1\rangle)\\ &-|HH\rangle|\widetilde{Ir}_1\widetilde{Ir}_1\rangle + |VV\rangle|\widetilde{Ir}_1\widetilde{Ir}_1\rangle\\ &-|HH\rangle|\widetilde{Er}_2\widetilde{Er}_2\rangle + |VV\rangle|\widetilde{Er}_2\widetilde{Er}_2\rangle\\ &+|HH\rangle|\widetilde{Ir}_2\widetilde{Ir}_2\rangle - |VV\rangle|\widetilde{Ir}_2\widetilde{Ir}_2\rangle\\ &+|HH\rangle|\widetilde{Er}_1\widetilde{Er}_1\rangle - |VV\rangle|\widetilde{Er}_1\widetilde{Er}_1\rangle)\end{aligned} \tag{13}$$

Table 1. The relation of the input states and the signatures of detectors in view of the detection time interval Δt.

	Input state	Detector signature	Δt
1	$\phi_P^+ \otimes \phi_S^+$	$D_1^{\pm} D_8^{\mp}$, $D_2^{\pm} D_7^{\mp}$, $D_3^{\pm} D_6^{\mp}$, $D_4^{\pm} D_5^{\mp}$	
2	$\phi_P^+ \otimes \phi_S^-$	$D_1^{\pm} D_2^{\pm}$, $D_3^{\pm} D_4^{\pm}$, $D_5^{\pm} D_6^{\pm}$, $D_7^{\pm} D_8^{\pm}$	
3	$\phi_P^- \otimes \phi_S^+$	$D_1^{\pm} D_8^{\pm}$, $D_2^{\pm} D_7^{\pm}$, $D_3^{\pm} D_6^{\pm}$, $D_4^{\pm} D_5^{\pm}$	
4	$\phi_P^- \otimes \phi_S^-$	$D_1^{\pm} D_2^{\mp}$, $D_3^{\pm} D_4^{\mp}$, $D_5^{\pm} D_6^{\mp}$, $D_7^{\pm} D_8^{\mp}$	
5	$\psi_P^+ \otimes \psi_S^+$	$D_1^{\pm} D_5^{\mp}$, $D_2^{\pm} D_6^{\mp}$, $D_3^{\pm} D_7^{\mp}$, $D_4^{\pm} D_8^{\mp}$	0
6	$\psi_P^+ \otimes \psi_S^-$	$D_1^{\pm} D_1^{\pm}$, $D_2^{\pm} D_2^{\pm}$, $D_3^{\pm} D_3^{\pm}$, $D_4^{\pm} D_4^{\pm}$	
		$D_5^{\pm} D_5^{\pm}$, $D_6^{\pm} D_6^{\pm}$, $D_7^{\pm} D_7^{\pm}$, $D_8^{\pm} D_8^{\pm}$	
7	$\psi_P^- \otimes \psi_S^+$	$D_1^{\pm} D_7^{\pm}$, $D_2^{\pm} D_8^{\pm}$, $D_3^{\pm} D_5^{\pm}$, $D_4^{\pm} D_6^{\pm}$	
8	$\psi_P^- \otimes \psi_S^-$	$D_1^{\pm} D_3^{\mp}$, $D_2^{\pm} D_4^{\mp}$, $D_5^{\pm} D_7^{\mp}$, $D_6^{\pm} D_8^{\mp}$	
9	$\phi_P^+ \otimes \psi_S^+$	$D_1^{\pm} D_6^{\mp}$, $D_2^{\pm} D_5^{\mp}$, $D_3^{\pm} D_8^{\mp}$, $D_4^{\pm} D_7^{\mp}$	
		$D_1^{\pm} D_4^{\mp}$, $D_2^{\pm} D_3^{\mp}$, $D_5^{\pm} D_8^{\mp}$, $D_6^{\pm} D_7^{\mp}$	
10	$\phi_P^+ \otimes \psi_S^-$	$D_1^{\pm} D_8^{\pm}$, $D_2^{\pm} D_7^{\pm}$, $D_3^{\pm} D_6^{\pm}$, $D_4^{\pm} D_5^{\pm}$	
		$D_1^{\pm} D_2^{\pm}$, $D_3^{\pm} D_4^{\pm}$, $D_5^{\pm} D_6^{\pm}$, $D_7^{\pm} D_8^{\pm}$	
11	$\phi_P^- \otimes \psi_S^+$	$D_1^{\pm} D_6^{\pm}$, $D_2^{\pm} D_5^{\pm}$, $D_3^{\pm} D_8^{\pm}$, $D_4^{\pm} D_7^{\pm}$	
		$D_1^{\pm} D_4^{\pm}$, $D_2^{\pm} D_3^{\pm}$, $D_5^{\pm} D_8^{\pm}$, $D_6^{\pm} D_7^{\pm}$	
12	$\phi_P^- \otimes \psi_S^-$	$D_1^{\pm} D_8^{\mp}$, $D_2^{\pm} D_7^{\mp}$, $D_3^{\pm} D_6^{\mp}$, $D_4^{\pm} D_5^{\mp}$	
		$D_1^{\pm} D_2^{\mp}$, $D_3^{\pm} D_4^{\mp}$, $D_5^{\pm} D_6^{\mp}$, $D_7^{\pm} D_8^{\mp}$	
13	$\psi_P^+ \otimes \phi_S^+$	$D_1^{\pm} D_7^{\mp}$, $D_2^{\pm} D_8^{\mp}$, $D_3^{\pm} D_5^{\mp}$, $D_4^{\pm} D_6^{\mp}$	t
		$D_1^{\pm} D_1^{\mp}$, $D_2^{\pm} D_2^{\mp}$, $D_3^{\pm} D_3^{\mp}$, $D_4^{\pm} D_4^{\mp}$	
		$D_5^{\pm} D_5^{\mp}$, $D_6^{\pm} D_6^{\mp}$, $D_7^{\pm} D_7^{\mp}$, $D_8^{\pm} D_8^{\mp}$	
14	$\psi_P^+ \otimes \phi_S^-$	$D_1^{\pm} D_7^{\pm}$, $D_2^{\pm} D_8^{\pm}$, $D_3^{\pm} D_5^{\pm}$, $D_4^{\pm} D_6^{\pm}$	
		$D_1^{\pm} D_1^{\pm}$, $D_2^{\pm} D_2^{\pm}$, $D_3^{\pm} D_3^{\pm}$, $D_4^{\pm} D_4^{\pm}$	
		$D_5^{\pm} D_5^{\pm}$, $D_6^{\pm} D_6^{\pm}$, $D_7^{\pm} D_7^{\pm}$, $D_8^{\pm} D_8^{\pm}$	
15	$\psi_P^- \otimes \phi_S^+$	$D_1^{\pm} D_5^{\pm}$, $D_2^{\pm} D_6^{\pm}$, $D_3^{\pm} D_7^{\pm}$, $D_4^{\pm} D_8^{\pm}$	
		$D_1^{\pm} D_3^{\pm}$, $D_2^{\pm} D_4^{\pm}$, $D_5^{\pm} D_7^{\pm}$, $D_6^{\pm} D_8^{\pm}$	
16	$\psi_P^- \otimes \phi_S^-$	$D_1^{\pm} D_5^{\mp}$, $D_2^{\pm} D_6^{\mp}$, $D_3^{\pm} D_7^{\mp}$, $D_4^{\pm} D_8^{\mp}$	
		$D_1^{\pm} D_3^{\mp}$, $D_2^{\pm} D_4^{\mp}$, $D_5^{\pm} D_7^{\mp}$, $D_6^{\pm} D_8^{\mp}$	

At the output ports, the two photons are detected with the polarization basis $\{|H\rangle, |V\rangle\}$ by using PBSs. The detection result of the hyperentangled Bell state $|\psi_P^+ \psi_S^- \chi_S^+\rangle_{AB}$ corresponds to one of the 16 cases

$$\{D_1^{\pm} D_1^{\pm},\ D_2^{\pm} D_2^{\pm},\ D_3^{\pm} D_3^{\pm},\ D_4^{\pm} D_4^{\pm},\ D_5^{\pm} D_5^{\pm},\ D_6^{\pm} D_6^{\pm},\ D_7^{\pm} D_7^{\pm},\ D_8^{\pm} D_8^{\pm}\} \tag{14}$$

which is the result 6 in **Table 1**. Here two photons are routed to one output port and detected simultaneously by the same detector, which requires photon number resolving detector. The function of photon number resolving detector can be substituted by putting a BS at each output port of PBS before the photon detector, which has also be adopted by Schuck et al.[25] The superscript "+" in Equation (14) indicates the output port associated with transmission port of the PBS (corresponding to $|H\rangle$ polarization state), and the superscript "−" indicates the output port associated with reflection port of the PBS (corresponding to $|V\rangle$ polarization state). For example, the term "$D_1^+ D_1^+$" means that two photons are transmitted through PBS of port D_1. The term "$D_1^- D_1^-$" means that two photons are reflected by PBS of port D_1.

To make the principle of complete HBSA assisted by time bin more clear, the analysis of hyperentangled Bell state $|\psi_P^+ \phi_S^+ \chi_S^+\rangle_{AB}$ is also discussed. The hyperentangled Bell state $|\psi_P^+ \phi_S^+ \chi_S^+\rangle_{AB}$ can be described as

$$\begin{aligned}|\Theta_2^0\rangle_{AB} &= \frac{1}{2\sqrt{2}}(|HV\rangle + |VH\rangle)_{AB} \otimes (|II\rangle + |EE\rangle)_{AB} \\ &\quad \otimes (|lr\rangle + |rl\rangle)_{AB}\end{aligned} \tag{15}$$

In this case, we can use the setup of complete HBSA assisted by time bin (shown in Figure 1) to distinguish this state with the same method as above. That is, two photons A and B are first sent into the device from the input ports of PBSs in Figure 1. After the two photons pass through PBSs, the hyperentangled Bell state of two-photon system AB evolves to

$$\begin{aligned}|\Theta_2^1\rangle_{AB} &= \frac{1}{2\sqrt{2}}(|HV\rangle + |VH\rangle) \otimes [|Il_A Il_B\rangle + |Ir_A Ir_B\rangle \\ &\quad + |El_A El_B\rangle + |Er_A Er_B\rangle]\end{aligned} \tag{16}$$

Second, the wave packets of two photons A and B are mixed on BSs, which will transform the hyperentangled state of two-photon system AB into

$$\begin{aligned}|\Theta_2^2\rangle_{AB} &= \frac{1}{4\sqrt{2}}[(Il_{1H}^{\dagger} + Il_{2H}^{\dagger})(Il_{1V}^{\dagger} - Il_{2V}^{\dagger}) \\ &\quad + (Ir_{1H}^{\dagger} + Ir_{2H}^{\dagger})(Ir_{1V}^{\dagger} - Ir_{2V}^{\dagger}) \\ &\quad + (El_{1H}^{\dagger} + El_{2H}^{\dagger})(El_{1V}^{\dagger} - El_{2V}^{\dagger}) \\ &\quad + (Er_{1H}^{\dagger} + Er_{2H}^{\dagger})(Er_{1V}^{\dagger} - Er_{2V}^{\dagger}) \\ &\quad + (Il_{1V}^{\dagger} + Il_{2V}^{\dagger})(Il_{1H}^{\dagger} - Il_{2H}^{\dagger}) \\ &\quad + (Ir_{1V}^{\dagger} + Ir_{2V}^{\dagger})(Ir_{1H}^{\dagger} - Ir_{2H}^{\dagger}) \\ &\quad + (El_{1V}^{\dagger} + El_{2V}^{\dagger})(El_{1H}^{\dagger} - El_{2H}^{\dagger}) \\ &\quad + (Er_{1V}^{\dagger} + Er_{2V}^{\dagger})(Er_{1H}^{\dagger} - Er_{2H}^{\dagger})]|0\rangle \\ &= \frac{1}{2\sqrt{2}}(Il_{1H}^{\dagger} Il_{1V}^{\dagger} - Il_{2H}^{\dagger} Il_{2V}^{\dagger} + Ir_{1H}^{\dagger} Ir_{1V}^{\dagger} \\ &\quad - Ir_{2H}^{\dagger} Ir_{2V}^{\dagger} + El_{1H}^{\dagger} El_{1V}^{\dagger} - El_{2H}^{\dagger} El_{2V}^{\dagger} \\ &\quad + Er_{1H}^{\dagger} Er_{1V}^{\dagger} - Er_{2H}^{\dagger} Er_{2V}^{\dagger})|0\rangle\end{aligned} \tag{17}$$

Third, the two photons are put into UI (shown in Figure 2) from the input ports of PBSs to introduce time bin. After passing through PBSs and experiencing the time delay t, the hyperentangled state of two-photon system can be transformed to

$$\begin{aligned}|\Theta_2^3\rangle_{AB} &= \frac{1}{2\sqrt{2}}(Il_{1H'}^{\dagger} El_{2V}^{\dagger} - Il_{2H'}^{\dagger} El_{1V}^{\dagger} + Ir_{1H'}^{\dagger} Er_{2V}^{\dagger} \\ &\quad - Ir_{2H'}^{\dagger} Er_{1V}^{\dagger} + El_{1H}^{\dagger} Il_{2V'}^{\dagger} - El_{2H}^{\dagger} Il_{1V'}^{\dagger} \\ &\quad + Er_{1H}^{\dagger} Ir_{2V'}^{\dagger} - Er_{2H}^{\dagger} Ir_{1V'}^{\dagger})|0\rangle\end{aligned} \tag{18}$$

After that, the wave packets of two photons pass through BSs, which will transform the hyperentangled state $|\Theta_2^3\rangle_{AB}$ to

$$\begin{aligned}|\Theta_2^4\rangle_{AB} = \frac{1}{4\sqrt{2}}[&(\widetilde{Il}^{\dagger}_{1H'} + \widetilde{El}^{\dagger}_{2H'})(\widetilde{Il}^{\dagger}_{1V} - \widetilde{El}^{\dagger}_{2V})\\ &-(\widetilde{Il}^{\dagger}_{2H'} + \widetilde{El}^{\dagger}_{1H'})(\widetilde{Il}^{\dagger}_{2V} - \widetilde{El}^{\dagger}_{1V})\\ &+(\widetilde{Ir}^{\dagger}_{1H'} + \widetilde{Er}^{\dagger}_{2H'})(\widetilde{Ir}^{\dagger}_{1V} - \widetilde{Er}^{\dagger}_{2V})\\ &-(\widetilde{Ir}^{\dagger}_{2H'} + \widetilde{Er}^{\dagger}_{1H'})(\widetilde{Ir}^{\dagger}_{2V} - \widetilde{Er}^{\dagger}_{1V})\\ &+(\widetilde{Il}^{\dagger}_{2H} - \widetilde{El}^{\dagger}_{1H})(\widetilde{Il}^{\dagger}_{2V'} + \widetilde{El}^{\dagger}_{1V'})\\ &-(\widetilde{Il}^{\dagger}_{1H} - \widetilde{El}^{\dagger}_{2H})(\widetilde{Il}^{\dagger}_{1V'} + \widetilde{El}^{\dagger}_{2V'})\\ &+(\widetilde{Ir}^{\dagger}_{2H} - \widetilde{Er}^{\dagger}_{1H})(\widetilde{Ir}^{\dagger}_{2V'} + \widetilde{Er}^{\dagger}_{1V'})\\ &-(\widetilde{Ir}^{\dagger}_{1H} - \widetilde{Er}^{\dagger}_{2H})(\widetilde{Ir}^{\dagger}_{1V'} + \widetilde{Er}^{\dagger}_{2V'})]|0\rangle\end{aligned} \tag{19}$$

In the final step, the polarization Hadamard operations are performed on the wave packets of two photons with HWPs, and the hyperentangled state becomes

$$\begin{aligned}|\Theta_2^5\rangle_{AB} = \frac{1}{4\sqrt{2}}(&-\widetilde{Il}^{\dagger}_{1H'}\widetilde{Il}^{\dagger}_{1V} + \widetilde{Il}^{\dagger}_{1V'}\widetilde{Il}^{\dagger}_{1H} + \widetilde{El}^{\dagger}_{2H'}\widetilde{El}^{\dagger}_{2V}\\ &-\widetilde{El}^{\dagger}_{2V'}\widetilde{El}^{\dagger}_{2H} + \widetilde{Il}^{\dagger}_{1H'}\widetilde{El}^{\dagger}_{2V} - \widetilde{Il}^{\dagger}_{1V'}\widetilde{El}^{\dagger}_{2H}\\ &-\widetilde{El}^{\dagger}_{2H'}\widetilde{Il}^{\dagger}_{1V} + \widetilde{El}^{\dagger}_{2V'}\widetilde{Il}^{\dagger}_{1H} + \widetilde{Il}^{\dagger}_{2H'}\widetilde{Il}^{\dagger}_{2V}\\ &-\widetilde{Il}^{\dagger}_{2V'}\widetilde{Il}^{\dagger}_{2H} - \widetilde{El}^{\dagger}_{1H'}\widetilde{El}^{\dagger}_{1V} + \widetilde{El}^{\dagger}_{1V'}\widetilde{El}^{\dagger}_{1H}\\ &-\widetilde{Il}^{\dagger}_{2H'}\widetilde{El}^{\dagger}_{1V} + \widetilde{Il}^{\dagger}_{2V'}\widetilde{El}^{\dagger}_{1H} + \widetilde{El}^{\dagger}_{1H'}\widetilde{Il}^{\dagger}_{2V}\\ &-\widetilde{El}^{\dagger}_{1V'}\widetilde{Il}^{\dagger}_{2H} - \widetilde{Ir}^{\dagger}_{1H'}\widetilde{Ir}^{\dagger}_{1V} + \widetilde{Ir}^{\dagger}_{1V'}\widetilde{Ir}^{\dagger}_{1H}\\ &+\widetilde{Er}^{\dagger}_{2H'}\widetilde{Er}^{\dagger}_{2V} - \widetilde{Er}^{\dagger}_{2V'}\widetilde{Er}^{\dagger}_{2H} + \widetilde{Ir}^{\dagger}_{1H'}\widetilde{Er}^{\dagger}_{2V}\\ &-\widetilde{Ir}^{\dagger}_{1V'}\widetilde{Er}^{\dagger}_{2H} - \widetilde{Er}^{\dagger}_{2H'}\widetilde{Ir}^{\dagger}_{1V} + \widetilde{Er}^{\dagger}_{2V'}\widetilde{Ir}^{\dagger}_{1H}\\ &+\widetilde{Ir}^{\dagger}_{2H'}\widetilde{Ir}^{\dagger}_{2V} - \widetilde{Ir}^{\dagger}_{2V'}\widetilde{Ir}^{\dagger}_{2H} - \widetilde{Er}^{\dagger}_{1H'}\widetilde{Er}^{\dagger}_{1V}\\ &+\widetilde{Er}^{\dagger}_{1V'}\widetilde{Er}^{\dagger}_{1H} - \widetilde{Ir}^{\dagger}_{2H'}\widetilde{Er}^{\dagger}_{1V} + \widetilde{Ir}^{\dagger}_{2V'}\widetilde{Er}^{\dagger}_{1H}\\ &+\widetilde{Er}^{\dagger}_{1H'}\widetilde{Ir}^{\dagger}_{2V} - \widetilde{Er}^{\dagger}_{1V'}\widetilde{Ir}^{\dagger}_{2H})|0\rangle\end{aligned} \tag{20}$$

The hyperentangled state $|\Theta_2^5\rangle_{AB}$ can also be described in the following form

$$\begin{aligned}|\Theta_2^5\rangle_{AB} = \frac{1}{4\sqrt{2}}(&-|H'V\rangle|\widetilde{Il}_1\widetilde{Il}_1\rangle + |V'H\rangle|\widetilde{Il}_1\widetilde{Il}_1\rangle\\ &+|H'V\rangle|\widetilde{El}_2\widetilde{El}_2\rangle - |V'H\rangle|\widetilde{El}_2\widetilde{El}_2\rangle\\ &+|H'V\rangle|\widetilde{Il}_1\widetilde{El}_2\rangle - |V'H\rangle|\widetilde{Il}_1\widetilde{El}_2\rangle\\ &-|H'V\rangle|\widetilde{El}_2\widetilde{Il}_1\rangle + |V'H\rangle|\widetilde{El}_2\widetilde{Il}_1\rangle\\ &+|H'V\rangle|\widetilde{Il}_2\widetilde{Il}_2\rangle - |V'H\rangle|\widetilde{Il}_2\widetilde{Il}_2\rangle\\ &-|H'V\rangle|\widetilde{El}_1\widetilde{El}_1\rangle + |V'H\rangle|\widetilde{El}_1\widetilde{El}_1\rangle\\ &-|H'V\rangle|\widetilde{Il}_2\widetilde{El}_1\rangle + |V'H\rangle|\widetilde{Il}_2\widetilde{El}_1\rangle\\ &+|H'V\rangle|\widetilde{El}_1\widetilde{Il}_2\rangle - |V'H\rangle|\widetilde{El}_1\widetilde{Il}_2\rangle\\ &-|H'V\rangle|\widetilde{Ir}_1\widetilde{Ir}_1\rangle + |V'H\rangle|\widetilde{Ir}_1\widetilde{Ir}_1\rangle\\ &+|H'V\rangle|\widetilde{Er}_2\widetilde{Er}_2\rangle - |V'H\rangle|\widetilde{Er}_2\widetilde{Er}_2\rangle\\ &+|H'V\rangle|\widetilde{Ir}_1\widetilde{Er}_2\rangle - |V'H\rangle|\widetilde{Ir}_1\widetilde{Er}_2\rangle\\ &-|H'V\rangle|\widetilde{Er}_2\widetilde{Ir}_1\rangle + |V'H\rangle|\widetilde{Er}_2\widetilde{Ir}_1\rangle\\ &+|H'V\rangle|\widetilde{Ir}_2\widetilde{Ir}_2\rangle - |V'H\rangle|\widetilde{Ir}_2\widetilde{Ir}_2\rangle\\ &-|H'V\rangle|\widetilde{Er}_1\widetilde{Er}_1\rangle + |V'H\rangle|\widetilde{Er}_1\widetilde{Er}_1\rangle\\ &-|H'V\rangle|\widetilde{Ir}_2\widetilde{Er}_1\rangle + |V'H\rangle|\widetilde{Ir}_2\widetilde{Er}_1\rangle\\ &+|H'V\rangle|\widetilde{Er}_1\widetilde{Ir}_2\rangle - |V'H\rangle|\widetilde{Er}_1\widetilde{Ir}_2\rangle)\end{aligned} \tag{21}$$

Here the time delay t satisfies the condition of constructive interference.

At the output ports, the two photons are detected in the polarization basis $\{|H\rangle, |V\rangle\}$ with PBSs. The detection event of the state $|\psi_P^+\phi_S^+\chi_S^+\rangle_{AB}$ corresponds to one of the 24 cases

$$\begin{aligned}&\{D_1^{\pm}D_7^{\mp},\ D_2^{\pm}D_8^{\mp},\ D_3^{\pm}D_5^{\mp},\ D_4^{\pm}D_6^{\mp},\ D_1^{\pm}D_1^{\mp},\ D_2^{\pm}D_2^{\mp},\\ &D_3^{\pm}D_3^{\mp},\ D_4^{\pm}D_4^{\mp},\ D_5^{\pm}D_5^{\mp},\ D_6^{\pm}D_6^{\mp},\ D_7^{\pm}D_7^{\mp},\ D_8^{\pm}D_8^{\mp}\}\end{aligned} \tag{22}$$

which is the result 13 in Table 1. Here two detectors are clicked one after another with time interval t.

Now, we have introduced all the steps of unambiguous discrimination of hyperentangled Bell state for two-photon system entangled in both polarization and momentum DOFs by analyzing the hyperentangled Bell states $|\psi_P^+\psi_S^-\chi_S^+\rangle_{AB}$ and $|\psi_P^+\phi_S^+\chi_S^+\rangle_{AB}$ as example. The other 14 hyperentangled Bell states can be analyzed by the setup of unambiguous discrimination of hyperentangled Bell state assisted by time bin (shown in Figure 1) as well (see supporting Information for details). The detection results of the 16 hyperentangled Bell states are shown in Table 1, where t signifies the time interval of photon-pair detection. According to the time interval, 16 hyperentangled Bell states are evenly classified into two groups. By comparison, we can see that the detection results of the hyperentangled Bell states in each group are distinct from each other. Since the parity information of both the momentum and polarization DOFs can be obtained resorting to time delay and auxiliary entangled state of another momentum DOF, the 16 hyperentangled Bell states can be discriminated completely after the product measurement, where the phase information of polarization and momentum DOFs can also be acquired in product measurement.

3. Discussion and Summary

We present a linear optical unambiguous discrimination scheme for two-photon systems entangled in both the polarization and momentum degrees of freedom (DOFs) assisted by time-bin. It is well known that distinguishing hyperentangled Bell state is to identify the parity information denoted by "ψ" or "ϕ" and phase information represented by "+" or "−" in each DOF. In our unambiguous discrimination scheme, time bin is introduced to read out the parity information of the momentum DOF with the different detection time intervals (0 or t), and the parity information of the polarization DOF is transferred to the auxiliary momentum DOF and read out by detection signatures in the auxiliary momentum DOF with the computation basis. The phase information of both the polarization and momentum DOFs can be read out by detection signatures in the polarization and momentum DOFs with the computation basis. Combined detection time intervals and detection signatures together, it is easy to distinguish the 16 orthogonal hyperentangled Bell states completely.

The UI with an optical time delay t is an essential element in our unambiguous discrimination scheme for introducing time bin, which requires the constructive interference to be met. That is, the time interval between the wave packets of input state is adjusted to satisfy the condition of constructive interference with $\omega t = 2m\pi$ (m is an integer), which has been demonstrated experimentally by Williams et al.[33] In their experiment, the photon pairs prepared by SPDC are sent into an interferometer with two time delays $t_0 = 5ns$ and $t_1 = 10ns$, and the time resolution for photon detectors is $4ns$. Correspondingly, the fiber lengths of the short and long arms of the interferometer are set to be $1m$ and $2m$ for the first loop and $2m$ and $4m$ for the second loop. If the time delays in the interferometer are multiples of the pulse period t_p, for instance, $t_0 = t_p$ and $t_1 = 2t_p$, the experiment can be improved. In addition, some other factors should also be considered in practice, such as drift in the interferometer during transmission, phase miscalibration, and imperfect state generation, which may decrease the efficiency of unambiguous discrimination scheme in experiment.

When the number of DOFs is n ($n \geq 2$) for the two-photon system in hyperentangled Bell state, our method is suitable as well. In the previous works [56–58], the number of distinguishable groups for hyperentangled Bell states in n DOFs has been calculated. That is, with linear optics, 4^n hyperentangled states in n DOFs can be separated into $x_0(n)$ groups. Here

$$x_0(n) = (2^n - 1) * 2 + 1 = 2^{n+1} - 1 \tag{23}$$

If k additional entangled states of additional DOFs are introduced, the number of distinguishable groups is

$$x_k = 2x_{k-1} - 2^{2k-1} \tag{24}$$

Here $k = 1, 2, 3, \ldots, n$. Therefore, $x_k(n) = 2^{n+k+1} - 2^{2k}$ groups out of 4^n states can be distinguished with k additional entangled states of additional DOFs. If $k = n$, the unambiguous distinction of 4^n hyperentangled Bell states can be realized. For example, with $k = 2$ auxiliary entangled states of additional DOFs, 16 hyperentangled Bell states can be distinguished completely. We get the same result with time bin and only one auxiliary entangled state of additional DOF. Therefore, in essence, the function of time bin in our scheme is equivalent to the one of an auxiliary entangled state of additional DOF. We can generalize this idea to n-DOF hyperentangled system. To distinguish 2^{n+k+1} hyperentangled Bell states in n DOFs, time bin and k ($k < n$) auxiliary entangled states of additional DOFs are required, and ($2^{n+k+2} - 2^{2(k+1)}$) groups out of 4^n hyperentangled Bell states can be differentiated with time bin and k auxiliary entangle states of additional DOFs. For example, if $n = 2$ and $k = 0$, the 16 hyperentangled Bell states can be divided into 12 groups, and 8 hyperentangled Bell states can be distinguished exactly via linear optics assisted by time bin, which can be applied to photon system without entanglement (such as in hyperteleportation and hyperentanglement swapping). If more than one time delay (or more than one auxiliary DOF similar to time bin) are introduced, even more auxiliary entangled states of additional DOFs may be saved in the construction of complete HBSA. By introducing time bin, the implementation of complete HBSA in n-DOF hyperentangled system can be simplified considerably.

In conclusion, we have achieved the full distinction of 16 orthogonal hyperentangled Bell states deterministically via linear optics assisted by time bin. In this scheme, the parity information of the momentum Bell state and polarization Bell state can be distinguished assisted by a time delay and an auxiliary fixed Bell state of another momentum DOF, respectively. The phase information of the polarization Bell state and momentum Bell state can be distinguished using the product measurement. Moreover, this method is also suitable for distinguishing hyperentangled Bell states in n ($n \geq 2$) DOFs that satisfy the condition of constructive interference with the linear optical elements. Therefore, this scheme provides a new way for distinguishing hyperentangled state with current technology, which shows the function of the auxiliary entangled state of additional DOF can be replaced by time bin. As the time bin is introduced in the unambiguous discrimination scheme instead of the entanglement initially prepared in photon system, the application of hyperentangled states discrimination via linear optics will be extended to other quantum information protocols using hyperentangled Bell state measurement for photon system without entanglement (e.g., hyperteleportation and hyperentanglement swapping) besides the ones using hyperentangled Bell state measurement for photon system with entanglement (e.g., hyperdense coding) in the future.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

linear optics, quantum information, time bin, unambiguous discrimination of hyperentangled Bell states

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